Comment on "Observation of a Push Force on the End Face of a Nanometer Silica Filament Exerted by Outgoing Light"

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The Abraham-Minkowski electromagnetic energy-momentum tensor problem has been on the agenda since about 1910. The recent experiment of She *et al.* [1] is in this connection of interest, as it shows how the radiation force from a low-intensity laser yields an inward push force on the end face of a vertical fiber.

But does this experiment measure electromagnetic momentum? In our opinion the answer is no. What is detected is merely the electromagnetic Abraham-Minkowski force density $\mathbf{f}^{AM} = -(\varepsilon_0/2)E^2\nabla n^2$ in the surface layer of the filament (or in other regions where n varies). This is not related to the electromagnetic momentum in itself. The electromagnetic force density is $\mathbf{f} = \mathbf{f}^{AM} + [(n^2-1)/c^2]\partial/\partial t(\mathbf{E}\times\mathbf{H})$, and electromagnetic momentum does not appear until the second term in this expression. This is the Abraham term. It is in principle measurable although it is usually small; moreover it simply fluctuates out when averaged over an optical period in a stationary beam.

For illustration, let us assume that a short laser pulse with energy $\mathcal{H}$ falls from vacuum towards the entrance surface of a free-standing fiber (we ignore gravity). If there is an antireflection film of refractive index $\sqrt{n}$ on the surface, $\mathcal{H}$ is the energy of the pulse in the medium also. The impulse imparted to the surface because of the surface force f^{AM} is $G_{surf} = \mathcal{H}(n-1)/c$, directed against the beam if $n > 1$. When the pulse leaves at the exit surface, a corresponding reverse impulse is imparted. In the Abraham case, one has to take into account the mechanical momentum G^A_{mech} caused by the Abraham term also. One finds $G^A_{mech} = \mathcal{H}(n^2-1)/nc$. The resulting longitudinal displacement of the fiber because of the sum $G_{surf} + G^A_{mech}$ becomes $\Delta x^A = (\mathcal{H}/c^2\mu)(n-1)$, where $\mu = M/L$ is the mass per unit length. As discussed on p. 189 in Ref. [2], $\Delta x^A \sim$ 1 pm or less, and is clearly non-observable.

In the present case, the fiber is fixed at the upper end. There will be a downward directed impulse imparted to the fiber at the lower end when the pulse leaves. It is very small: taking the flux to be 10 mW and the pulse duration to be 270 ms, we get $\mathcal{H} = 2.7$ mJ resulting in $G_{surf} = 4.5$ pN·s if $n = 1.5$. Because of elasticity, there will be an upward directed recoil in the fiber. (Sideways motion may result from non-axisymmetric elastic conditions.)

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We propose finally a modification of the experiment that might be capable of detecting the Abraham force after all (cf. also p. 191 of [2]): Let a long fiber of length L be wound up on a drum of radius R and of small weight. Suspend the drum as a vertical torsional pendulum such that it can oscillate about the z axis with an eigenfrequency ω_0. Then send an *intensity modulated* optical wave through the fiber, such that its harmonic component has the same frequency ω_0. We take the incident energy flux in vacuum to be $P^{(i)} = P_0\cos^2(\frac{1}{2}k_0x - \frac{1}{2}\omega_0 t)$, where $k_0 = \omega_0/c$, x is the longitudinal coordinate, and P_0 is the unmodulated energy flux averaged over an optical period. With antireflection films on the end surfaces we obtain in the Abraham case a longitudinal force $F^A = P_0[(n-1)/cn]\sin(\frac{1}{2}nk_0L)\sin(\frac{1}{2}nk_0L - \omega_0 t)$, whereas in the Minkowski case $F^M = -nF^A$. These forces give rise to measurable axial torques N_z on the drum. Assuming the sheet of fiber on the drum to be thin we obtain, when setting $\sin(\frac{1}{2}nk_0L) \approx \frac{1}{2}nk_0L$, in the Abraham case $N_z^A = [(n-1)/2c^2]RLP_0\omega_0\sin(\frac{1}{2}nk_0L - \omega_0 t)$. In the Minkowski case, $N_z^M = -nN_z^A$. The two predictions are thus quite different.

For definiteness,assume that a YAG laser at 1.06 μm produces the incident beam. Assume that a high power of $P_0 = 1$ kW can be transmitted through the fiber, and neglect any losses. Then, with $L = 100$ m, $n = 1.5$, $R = 10$ cm, $\omega_0 = 10$ s^{-1}, we obtain for the predicted torque amplitudes $N_z^A = 2.8\times 10^{-13}$ Nm, $N_z^M = 4.2\times 10^{-13}$ Nm.

The above amplitudes are less than those of Ref. [3] (10^{-12} Nm), but of the same order of magnitude as in Ref. [4]. Actually, they are greater than those in the classic experiment of Ref. [5] (10^{-16} Nm). Realization of our proposed experiment appears difficult but not impossible.

Finally, it should be mentioned that our Einstein-box argument above implicitly assumed wide lateral dimensions for the pulse. Cf. also the Comment of Mansuripur on this point [6].

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